

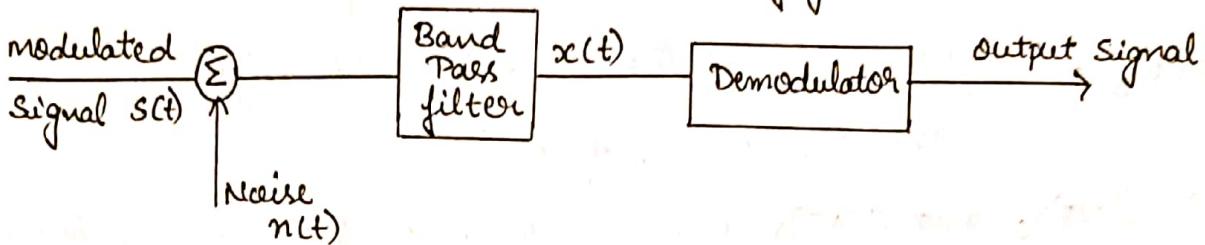
Module - 4

Noise in Analog Modulation

Noise performance of analog modulation system is evaluated by considering receiver model.

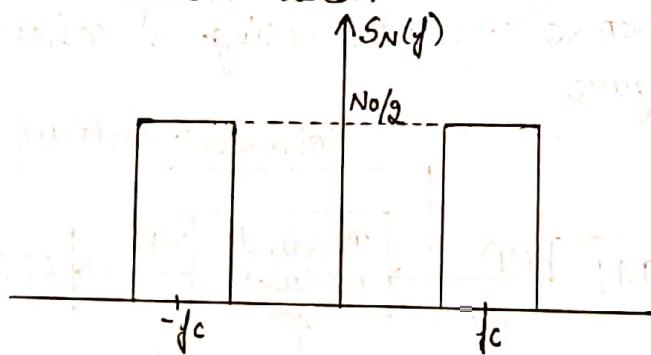
Receiver Model:

Receiver model is as shown in figure



In the above figure, $S(t)$ is the modulated signal and $n(t)$ is the noise signal. Signal $n(t)$ is known as front end receiver noise. The receiver input signal is the sum of $S(t)$ and $n(t)$.

$S(t) + n(t)$ passes through the Band pass filter, the bandwidth of BPF is equal to the transmission band width of the modulated signal. Demodulator used in the model depends on the type of modulation used.



Idealized characteristic of bandpass filtered noise

- Let $N_0/2$ is power spectral density [PSD] of noise $n(t)$ for both positive and negative frequency.
- N_0 is the average noise power per unit bandwidth.
- f_c is midband frequency equal to center frequency.
- $f_c \gg B_T$ so, filtered noise $n(t)$ as a narrow band noise and it is defined in canonical form by,

$$\langle n(t) \rangle = n_I(t) \cos(\omega \pi f t) - n_Q(t) \sin(\omega \pi f t)$$

$n_I(t)$ is inphase noise component & $n_Q(t)$ is quadrature noise component.
The filtered signal $x(t)$ at input of demodulator is

$$x(t) = s(t) + n(t)$$

- Channel Signal to noise ratio is given by,

$$(SNR)_c = \frac{\text{Average power of modulated signal}}{\text{Average power of noise in the message Bandwidth}}$$

- Output Signal to noise ratio is given by,

$$(SNR)_o = \frac{\text{Average power of the demodulated signal}}{\text{Average power of noise}}$$

Both $(SNR)_c$ and $(SNR)_o$ are measured at the receiver side.

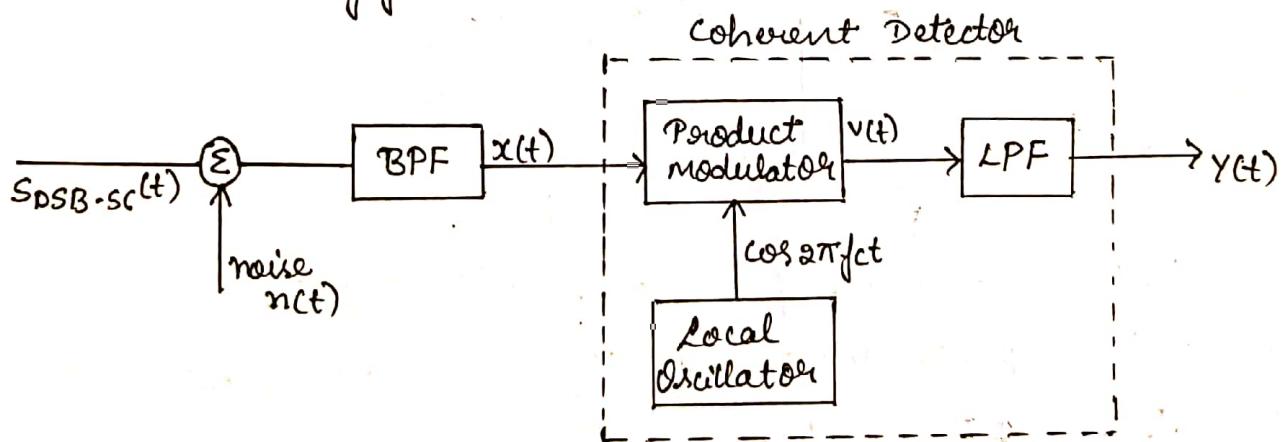
- Figure of merit for the receiver is given by,

$$\langle FOM = \frac{(SNR)_o}{(SNR)_c} \rangle$$

Higher the value of FOM, better the performance of the receiver. The value of FOM also depends on the type of modulation used.

Noise in DSB-SC Receiver:

The model of DSB-SC receiver using a coherent detector is shown in figure



- The filtered signal $x(t) = S_{DSB-SC}(t) + n(t)$ is applied to the coherent detector.
- In coherent detector, $x(t)$ is multiplied with a locally generated wave $\cos 2\pi f_c t$ using product modulator.
- The output of product modulator $v(t)$ is filtered using LPF to get output.

$$x(t) = S_{DSB-SC}(t) + m(t) \cos(\omega t)$$

$$= A_m m(t) \cos(\omega t)$$

Noise-band noise level is given by

$$n_r(t) = n_i(t) \cos(\omega t) - m(t) \sin(\omega t)$$

$(SNR)_c = \frac{\text{Average power of modulated signal}}{\text{Average power of noise in message BW}}$

$$\text{Average power of modulated signal} = \frac{(A_m m(t))^2}{2} = \frac{A_m^2 m^2(t)}{2}$$

$$= \frac{A_m^2 P}{2} \quad \because [m^2(t) = P]$$

where, P = average power of message.

Average power of noise in message bandwidth = N_{BW}

$$(SNR)_c = \frac{A_m^2 P / 2}{N_{BW}} = \frac{A_m^2 P}{2 N_{BW}}$$

$(SNR)_o = \frac{\text{Average power of demodulated signal}}{\text{Average power of noise}}$

From the DSB-SC receiver model

$$x(t) = S_{DSB-SC}(t) + n_i(t) \\ = A_m m(t) \cos(\omega t) + n_i(t) \cos(\omega t) - n_i(t) \sin(\omega t)$$

Then,

$$v(t) = x(t) \cos(2\pi f_c t)$$

$$v(t) = A_m m(t) \cos^2(\omega t) + n_i(t) \cos^2(\omega t) - n_i(t) \sin(\omega t) \cos(2\pi f_c t)$$

$$v(t) = A_m m(t) \left[\frac{1 + \cos 4\pi f_c t}{2} \right] + n_i(t) \left[\frac{1 + \cos 4\pi f_c t}{2} \right] - n_i(t) \frac{\sin(\omega t) \cos(2\pi f_c t)}{2}$$

$$v(t) = \frac{A_m m(t)}{2} + \frac{A_m m(t)}{2} \cos(4\pi f_c t) + \frac{n_i(t)}{2} + \frac{n_i(t)}{2} \cos(4\pi f_c t) - n_i(t) \frac{\sin(\omega t) \cos(2\pi f_c t)}{2}$$

When $v(t)$ passes through LPF, output $y(t)$ is

$$y(t) = \frac{A_m m(t)}{2} + \frac{n_i(t)}{2}$$

$$\text{Demodulated signal} = \frac{A c m(t)}{2}, \quad \text{Noise term} = \frac{n_I(t)}{2}$$

$$(SNR)_0 = \frac{\left(\frac{A c m(t)}{2}\right)^2}{\frac{1}{2} N_{\text{OW}}} = \frac{A c^2 m^2(t)}{4 \cdot \frac{1}{2}} \times \frac{2'}{N_{\text{OW}}} = \frac{A c^2 P}{2 N_{\text{OW}}}$$

$$(SNR)_0 = \frac{A c^2 P}{2 N_{\text{OW}}}$$

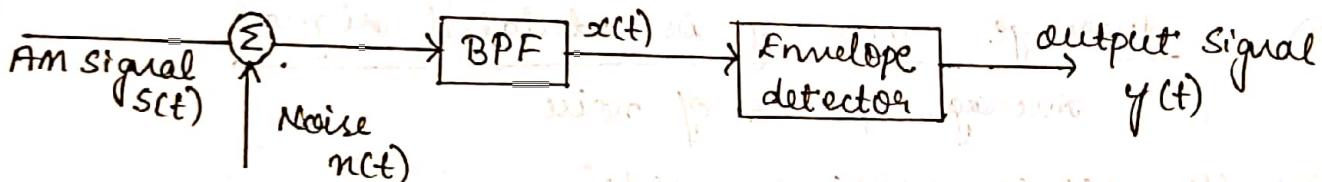
Then,

$$FOM = \frac{(SNR)_0}{(SNR)_C} = \frac{\frac{A c^2 P}{2 N_{\text{OW}}}}{\frac{A c^2 P}{2 N_{\text{OW}}}} = 1$$

$$\boxed{FOM = 1}$$

Noise in AM Receivers:

The model of AM receiver using envelop detector as demodulator is shown in figure.



The figure shows the model of an AM receiver, which used envelop detector for demodulation. The input signal $s(t)$ and noise $n(t)$ are added and given to BPF to make narrow band noise $n(t)$. So filtered signal is $x(t) = s(t) + n(t)$. The Envelop detector produces the required AM demodulated signal $y(t)$.

The i/p signal $s(t)$ in an AM modulated wave is given by

$$s_{\text{AM}}(t) = A_c [1 + k_m(t)] \cos(2\pi f_c t)$$

where, $A_c \cos 2\pi f_c t$ is the carrier wave
 $m(t)$ is modulating signal

k_m is constant that determines the modulation index.
Noise is given by,

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$(SNR)_c = \frac{\text{Avg power of } s_{AM}(t)}{\text{Avg power of } n(t) \text{ in message BW}}$

$$= \frac{\{A_c(1+K_a m(t))\}^2/2}{2N_0 W} = \frac{A_c^2 [1+K_a^2 m^2(t)]}{2 N_0 W}$$

$$(SNR)_c = \frac{A_c^2 [1+K_a^2 P]}{2 N_0 W} \quad : (m^2(t) \approx P)$$

To evaluate $(SNR)_o$

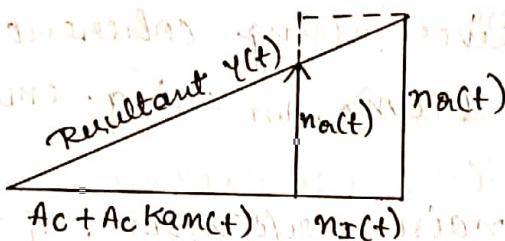
The signal $x(t)$ applied to the envelop detector in the receiver model.

$$x(t) = s_{AM}(t) + n(t)$$

$$x(t) = [A_c + A_c K_a m(t)] \cos \omega_f t + n_I(t) \cos 2\omega_f t - n_O(t) \sin 2\omega_f t$$

$$x(t) = [A_c + A_c K_a m(t) + n_I(t)] \cos \omega_f t - n_O(t) \sin \omega_f t$$

The phasor diagram for signal $x(t)$ is



From this phasor diagram, the receiver output can be obtained as

$$y(t) = \text{Envelope of } x(t)$$

$$y(t) = \sqrt{[A_c + A_c K_a m(t) + n_I(t)]^2 + [n_O(t)]^2}$$

when average power of carrier is large compared to average noise power. i.e $[A_c + A_c K_a m(t)]$ will be large compared with noise components $n_I(t)$ & $n_O(t)$.

$y(t)$ is approximated as

$$y(t) \approx A_c + A_c K_a m(t) + n_I(t)$$

The DC component A_c is removed by blocking capacitor, then

$$y(t) \approx A_c K_a m(t) + n_I(t)$$

$$(SNR)_0 = \frac{[AcKa_m(t)]^2}{2N_0} = \frac{Ac^2 Ka^2 P}{2N_0}$$

The above $(SNR)_0$, AM is valid only if two conditions are satisfied.

- i) The average noise power is small compared to the average carrier power at the envelope detector input.
- ii) The amplitude sensitivity K_a is less than or equal to 100%

$$FOM = \frac{(SNR)_0}{(SNR)_C} = \frac{Ac^2 Ka^2 P}{2N_0} / \frac{Ac^2 [1+Ka^2 P]}{2N_0}$$

$$= \frac{Ac^2 Ka^2 P}{2N_0} \times \frac{2N_0}{Ac^2 [1+Ka^2 P]}$$

$$\boxed{FOM = \frac{Ka^2 P}{1+Ka^2 P}}$$

Note:

FOM of a DSB-SC receiver using coherent detection is always unity, whereas FOM_{AM} using envelop detector is less than unity.

In other words, the noise performance of AM receiver is always inferior to that of DSB-SC receiver.

FOM of single tone AM Receiver:

For single tone, $m(t) = Am \cos 2\pi f_m t$

$$\therefore \text{avg power of } m(t) = P = \frac{Am^2}{2}$$

$$\text{W.K.T } (FOM)_{AM} = \frac{Ka^2 P}{1+Ka^2 P}$$

$$(FOM)_{STAM} = \frac{\frac{Ka^2 Am^2}{2}}{1+\frac{Ka^2 Am^2}{2}} = \frac{\frac{Ka^2 Am^2}{2}}{\frac{2+Ka^2 Am^2}{2}}$$

$$(FOM)_{STAM} = \frac{(KaAm)^2}{2+(KaAm)^2}$$

$$\left\langle (FOM)_{STAM} = \frac{u^2}{2+u^2} \right\rangle$$

$$\therefore [u = KaAm \text{ in AM}]$$

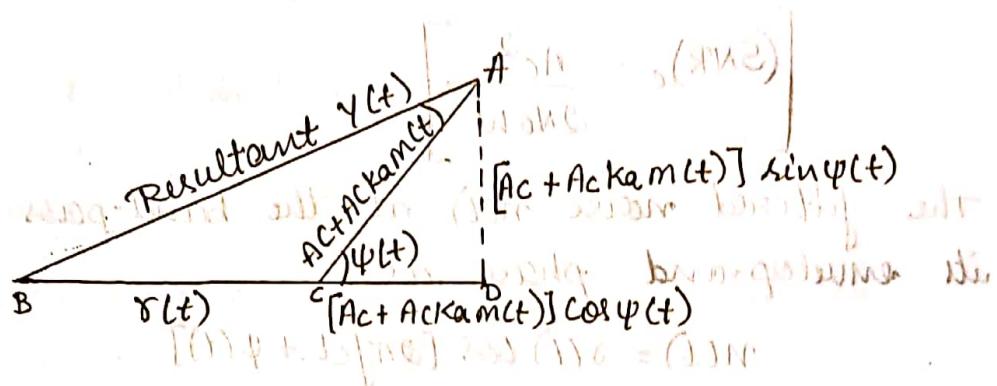
threshold effect:

when the carrier to noise ratio i.e $(CSNR)_C$ at the receiver input is small compared with unity, the noise term dominates and the performance of envelop detector changes.

In this case, $n(t)$ is represented in terms of its envelope $\tau(t)$ and phase $\psi(t)$ is

$$n(t) = \tau(t) \cos [\omega \tau t + \psi(t)]$$

The corresponding phasor diagram is constructed with reference to $\tau(t)$ as shown, since noise dominates.



$$Y(t) = \sqrt{\{r(t) + [Ac + Ackam(t)] \cos \psi(t)\}^2 + \{[Ac + Ackam(t)] \sin \psi(t)\}^2}$$

here $r(t) \gg Ac$, so neglecting quadrature component of signal, we get O/P of detector is

$$y(t) \approx r(t) + [Ac + Ackam(t)] \cos \psi(t)$$

$$y(t) \approx r(t) + Ac \cos \psi(t) + Ackam(t) \cos \psi(t)$$

From the above expression, when the carrier to noise ratio is low, the detector output has no component strictly proportional to the message signal $m(t)$.

The last term of the $y(t)$ i.e $Ackam(t) \cos \psi(t)$ contains $m(t)$ multiplied by noise in the form of $\cos \psi(t)$.

Thus, the loss of information/message in an envelope detector that operates at a low carrier to noise ratio is referred to as the threshold effect.

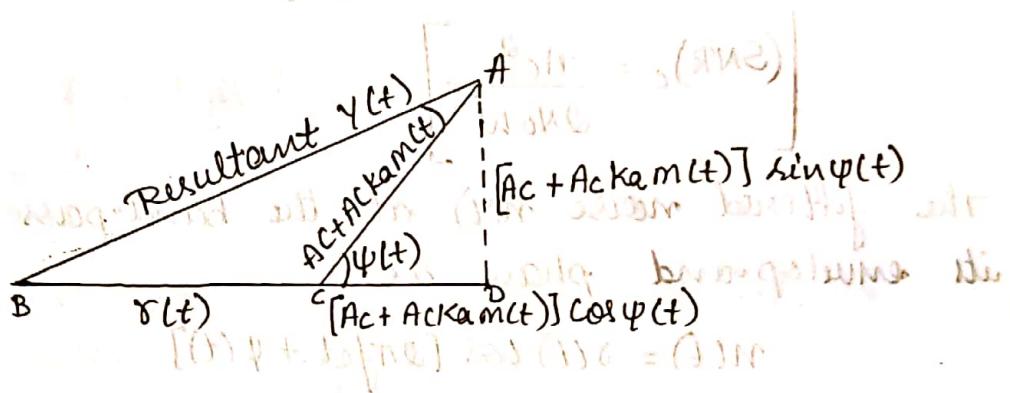
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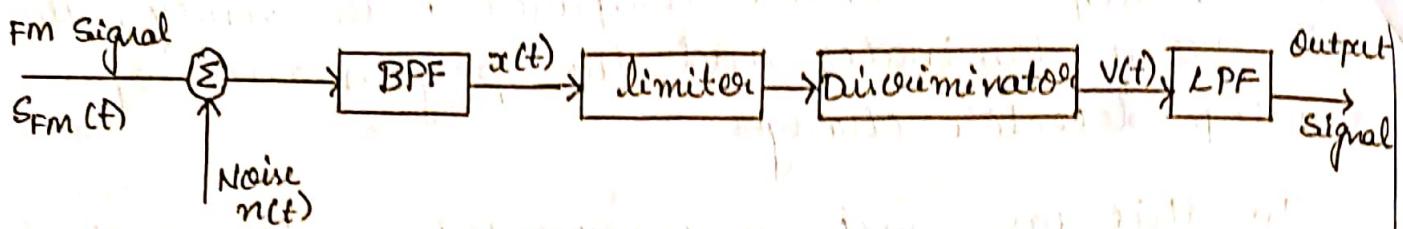
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Thus, the loss of information/message in an envelope detector that operates at a low carrier to noise ratio is referred to as the threshold effect.

Noise in FM Receiver!

FM receiver model is shown in figure



The incoming FM signal $S_{FM}(t)$ is given by

$$S_{FM}(t) = A_c \cos [\omega \pi f_c t + \omega \pi k_f \int_0^t m(\tau) d\tau] \quad (1)$$

$$S_{FM}(t) = A_c \cos [\omega \pi f_c t + \phi(t)]$$

where $\phi(t) = \omega \pi k_f \int_0^t m(\tau) d\tau$

$$[\text{SNR}]_c = \frac{A_c^2}{2 N_0 W}$$

the filtered noise $n(t)$ at the band-pass filter output in its envelop and phase as

$$n(t) = r(t) \cos [\omega \pi f_c t + \psi(t)]$$

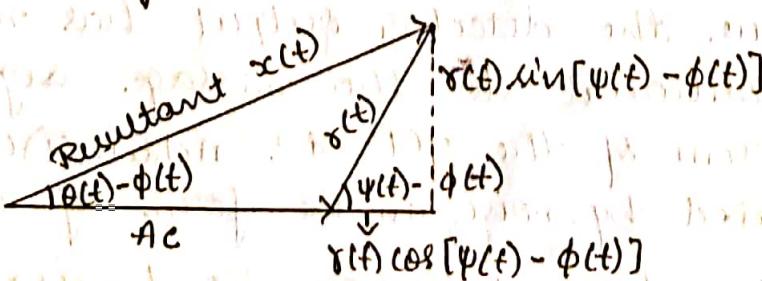
Where the envelope $r(t) = \sqrt{n_x^2(t) + n_\phi^2(t)}$

and the phase is $\psi(t) = \tan^{-1} \left[\frac{n_\phi(t)}{n_x(t)} \right]$

From the above figure, $x(t) = S_{FM}(t) + n(t)$

$$x(t) = A_c \cos [\omega \pi f_c t + \phi(t)] + r(t) \cos [\omega \pi f_c t + \psi(t)]$$

phasor diagram of $x(t)$ is shown along with noise



$$\tan [\phi(t) - \psi(t)] = \frac{r(t) \sin [\psi(t) - \phi(t)]}{A_c + r(t) \cos [\psi(t) - \phi(t)]}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\phi(t) = \phi(t) + \tan \left[\frac{\delta(t) \sin(\psi(t) - \phi(t))}{A_c + \gamma(t) \cos(\psi(t) - \phi(t))} \right]$$

$$\theta(t) = \phi(t) + \tan \left[\frac{\delta(t) \sin(\psi(t) - \phi(t))}{A_c + \gamma(t) \cos(\psi(t) - \phi(t))} \right]$$

Assume signal to noise ratio at the discriminator input to be much larger than unity
i.e. $A_c \gg \gamma(t)$

$$A_c + \gamma(t) \cos(\psi(t) - \phi(t)) \approx A_c$$

$$\theta(t) = \phi(t) + \tan \left[\frac{\delta(t) \sin(\psi(t) - \phi(t))}{A_c} \right]$$

$$\theta(t) = \underbrace{2\pi k_f \int_0^t m(\tau) d\tau}_{\text{Signal term}} + \tan \left[\frac{\delta(t) \sin(\psi(t) - \phi(t))}{A_c} \right] \underbrace{\text{Noise term}}$$

The discriminator output $v(t)$ is proportional to derivative of $\theta(t)$, i.e. $\frac{d\theta(t)}{dt}$

$$\text{i.e. } v(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$v(t) = \frac{1}{2\pi} \frac{d}{dt} \left\{ 2\pi k_f \int_0^t m(\tau) d\tau + \frac{\delta(t) \sin(\psi(t) - \phi(t))}{A_c} \right\}$$

$$v(t) = \frac{1}{2\pi} 2\pi k_f m(t) + \frac{1}{2\pi A_c} \frac{d}{dt} [\delta(t) \sin(\psi(t) - \phi(t))]$$

$$v(t) = k_f m(t) + n_d(t)$$

Thus output of the discriminator consists of original modulating signal $m(t)$ multiplied by scaling factor k_f , plus an additional noise component $n_d(t)$.

The average power of output noise is given

$$\text{by } \frac{2}{3} \frac{N_{\text{avg}}^3}{A_c^2}$$

$$\text{Thus } [\text{SNR}]_o = \frac{[K_f m(t)]^2}{\frac{2}{3} \frac{N_0 W^3}{A_C^2}}$$

$$[\text{SNR}]_o = \frac{K_f^2 P_3 A_C^2}{2 N_0 W^3}$$

$$\text{FOM} = \frac{[\text{SNR}]_o}{[\text{SNR}]_c} = \frac{3 A_C^2 K_f^2 P}{2 N_0 W^3} \times \frac{\omega / N_0 W}{A_C^2}$$

$$\boxed{\text{FOM} = \frac{3 K_f^2 P}{\omega^2}}$$

$$(\text{FOM})_{\text{FM}} \propto \left(\frac{K_f}{\omega}\right)^2 \propto \left(\frac{\Delta f}{\omega}\right)^2 \propto \beta^2$$

w.k.t frequency deviation Δf is proportional to the frequency sensitivity K_f of the modulator.

FOM for single tone FM Receiver:

For single tone $m(t) = A_m \cos \omega_f t$

Average power of $m(t) = P = A_m^2 / 2$

$$\text{w.k.t. } (\text{FOM})_{\text{FM}} = \frac{3 K_f^2 P}{\omega^2}$$

$$= \frac{3 K_f^2 A_m^2}{2 \omega^2} = \frac{3 (K_f A_m)^2}{2 \omega^2} = \frac{3}{2} \left(\frac{\Delta f}{\omega}\right)^2$$

$$\boxed{(\text{FOM})_{\text{FM}} = \frac{3}{2} \beta^2}$$

Capture Effect: When two or more signals are present at the input of the receiver, the output signal is affected by the stronger signal.

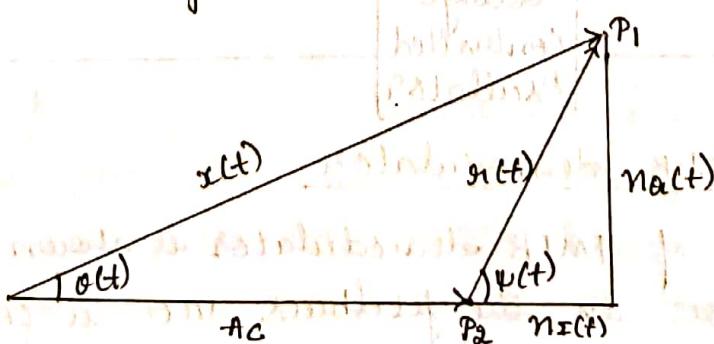
- In the frequency modulation, the signal can be affected by another frequency modulated signal whose frequency content is close to the carrier frequency of the desired FM wave.

receiver may lock to interference signal and suppresses the desired FM signal, when interference signal is stronger than the desired signal.

- When the strength of the desired signal and the interference signal are nearly equal, the receiver fluctuates back and forth between them, in this case receiver locks interference signal for some time and desired signal for the some time. This Phenomenon is known as the capture effect.

FM Threshold Effect:

- The output signal-to-noise ratio of an FM signal indicated by the equation i.e $[SNR]_o = \frac{3 A_c^2 K_f^2 P}{2 N_o W^3}$ is valid only if the carrier-to-noise ratio, measured at the discriminator input is high compared with unity.
- If the input noise power increases, the carrier-to-noise ratio decreases and receiver breaks.
- Initially, individual clicks are heard in the receiver output, and as the carrier-to-noise ratio decreases still further, the clicks rapidly merge into a crackling or sputtering sound.
- The threshold effect is defined as the minimum carrier to noise ratio that gives the output signal to noise ratio not less than the value predicted by the usual signal to noise formula assuming a small noise power.



Representation of equation using phasor diagram

The condition required to occur clicks are

conditions for positive clicks:

$$r(t) > A_c$$

$$\psi(t) < \pi < \psi(t) + d\psi(t)$$

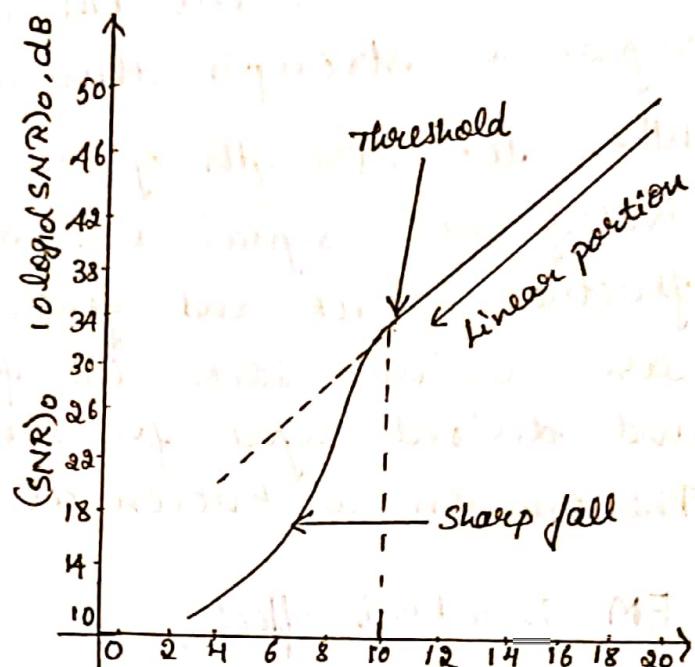
$$\frac{d\psi(t)}{dt} > 0$$

conditions for negative clicks:

$$r(t) > A_c$$

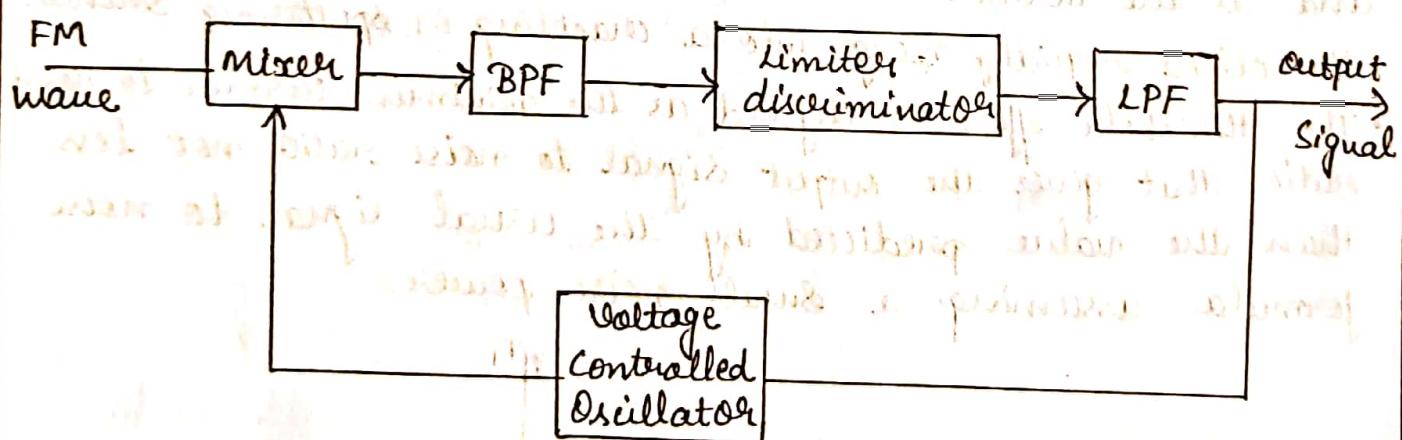
$$\psi(t) > -\pi > \psi(t) + d\psi(t)$$

$$\frac{d\psi(t)}{dt} < 0$$



FM Threshold Reduction:

In specific applications such as space communication lesser threshold in a FM receiver is required. For such applications, FM threshold can be reduced by using FM demodulator with negative feedback known as FMFB demodulator.



FMFB demodulator

Block diagram of FMFB demodulator is shown in above fig. From the figure the feedback VCO is connected and is controlled by the demodulated signal.

(7)

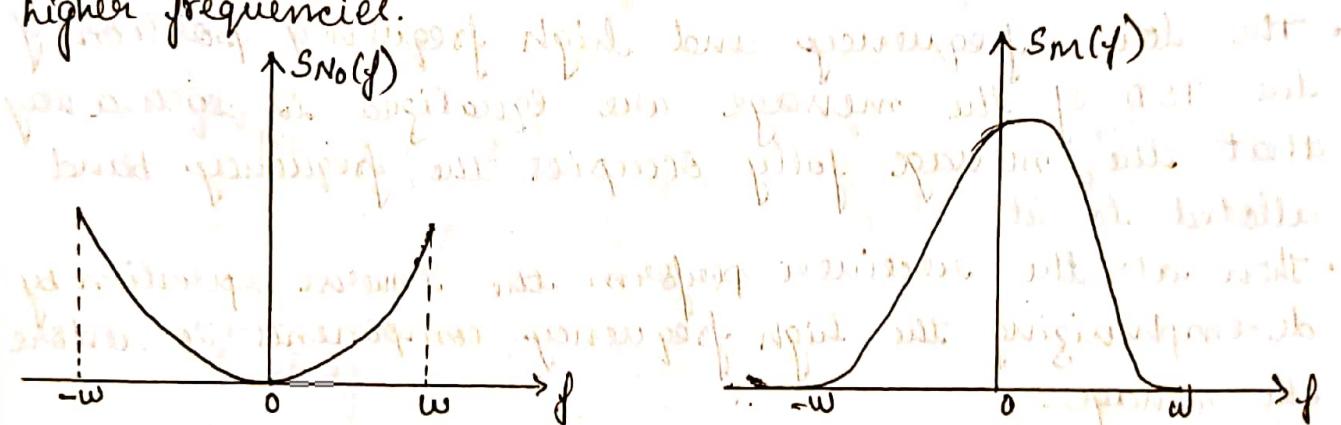
FMFB demodulator is essentially a tracking filter that can track only the slowly varying frequency of WBFM and consequently it responds only to a narrow band of noise centered about the instantaneous carrier frequency. As a result, FMFB receivers allow a threshold extension.

Like the FMFB demodulator, the PLL is also a tracking filter and hence it also provides threshold extension.

Pre-Emphasis and De-Emphasis in FM:

The power spectral density of the noise at the receiver output is shown in fig (a), it increases rapidly with frequency.

The power spectral density of a typical message source is shown in fig (b), it usually falls off appreciably at higher frequencies.



fig(a): PSD of output noise

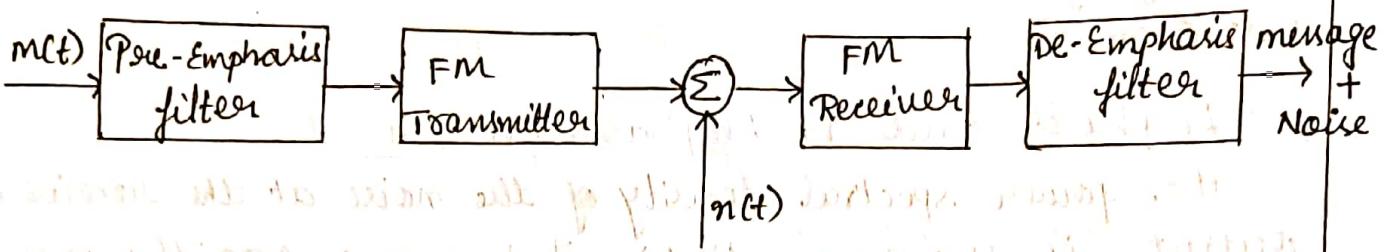
fig(b): PSD of message

Signal.

Thus at $f = \pm w$, the relative spectral density of the message is quite low, whereas that of the output noise is high in comparison. The message signal is not using the frequency band allowed to it in an efficient manner.

i) One way of improving the noise performance of the system is to slightly reduce the bandwidth of the LPF to reject large amount of noise power while losing only a small amount of message power. But this approach is not satisfactory because the distortion of the message.

ii) A more satisfactory approach is the use of pre-emphasis in the transmitter and de-emphasis in the receiver as shown in below figure



- In this method, we artificially emphasize the high frequency components of the message signal prior to modulation in the transmitter before the noise is introduced in the receiver.
- The low frequency and high frequency portions of the PSD of the message are equalized in such a way that the message fully occupies the frequency band allotted to it.
- Then at the receiver perform the inverse operation by de-emphasizing the high frequency components to restore the message.
- In this process, the high frequency noise is reduced thereby effectively increasing the output SNR of the system.

Pre-emphasis has the transfer function

$$H_{\text{pre}}(f) = k [1 + j \frac{f}{f_1}]$$

The corresponding de-emphasis network in the receiver will have a transfer function

$$H_{\text{de}}(f) = \frac{1}{H_{\text{pre}}(f)} = \frac{1}{k [1 + j \frac{f}{f_1}]}$$

constant K is chosen such that the average power of the pre-emphasized modulating signal be the same as the average power of the original modulating signal.

Derivation of improvement in SNR due to pre-emphasis and de-emphasis

Note: To derive 'K'

Let the power spectral density of the original modulating signal be

$$S_m(f) = \begin{cases} \frac{1}{1 + (\delta/f_1)^2} & \text{if } |f| < w \\ 0 & \text{elsewhere} \end{cases}$$

Average power of pre-emphasized signal = Average power of the original signal.

$$\int_{-\infty}^{\infty} |H(f)|^2 S_m(f) df = \int_{-\infty}^{\infty} S_m(f) df$$

$$\int_{-\infty}^{\infty} K^2 \left(1 + \frac{f^2}{f_1^2}\right)^{-1} df = \int_{-\infty}^{\infty} \frac{1}{1 + (\delta/f_1)^2} df$$

$$\int_{-\infty}^{\infty} K^2 df = \int_{-\infty}^{\infty} \frac{1}{1 + (\delta/f_1)^2} df$$

$$K^2 \left[f \right]_{-\infty}^{\infty} = \int_{-\infty}^{\infty} \frac{f_1 du}{1 + u^2}$$

$$\begin{aligned} \frac{\delta}{f_1} &= u \\ \delta &= u f_1 \\ du &= f_1 du \end{aligned}$$

$$2\omega K^2 = f_1 \left[\tan^{-1} \left(\frac{\delta}{f_1} \right) \right]_{-\infty}^{\infty}$$

$$2\omega K^2 = 2f_1 \tan^{-1} \left(\frac{\omega}{f_1} \right)$$

$$K^2 = \frac{f_1}{\omega} \tan^{-1} \left[\frac{\omega}{f_1} \right]$$

The improvement in SNR^{OP} due to pre-emphasis and de-emphasis in the transmitter and receiving end as

$$I = \frac{\text{Average output noise power without pre-emphasis \& de-emphasis}}{\text{Average output noise power with pre-emphasis and de-emphasis}}$$

$$\text{The average output noise power without pre-emphasis and de-emphasis} = \frac{2}{3} \frac{N_0 W^3}{A_C^2}$$

$$\text{Average output noise power with pre-emphasis and de-emphasis filter is given by, } \int_{-\omega}^{\omega} S_{NO}(f) |H_{de}(f)|^2 df$$

$$\text{where, } S_{NO}(f) = \frac{N_0 f^2}{A_C^2}$$

$$= \int_{-\omega}^{\omega} \frac{N_0 f^2}{A_C^2} \cdot \frac{1}{K^2 [1 + (\theta/f_1)^2]} df$$

Hence

$$I = \frac{\frac{2}{3} \frac{N_0 W^3}{A_C^2}}{\frac{N_0}{A_C^2 K^2} \int_{-\omega}^{\omega} f^2 \left[\frac{1}{1 + (\theta/f_1)^2} \right] df}$$

$$I = \frac{\frac{2}{3} \frac{W^3}{K^2}}{\int_{-\omega}^{\omega} \frac{f^2}{1 + (\theta/f_1)^2} df}$$

$$\text{Since } \int \frac{x^2 dx}{a^2 + b^2 x^2} = \frac{x}{b^2} - \frac{a^2}{b^2} \tan^{-1} \left(\frac{bx}{a} \right)$$

$$I = \frac{\frac{2}{3} \frac{W^3}{K^2}}{\int_{-\omega}^{\omega} \left[\frac{f^2}{1 + (\theta/f_1)^2} - \frac{\theta^2}{f_1^2} \tan^{-1} \left(\frac{\theta/f_1}{1} \right) \right] df}$$

$$I = \frac{\omega^3}{\frac{3k_2}{\omega} [\omega f_1^2 - f_1^3 \tan(\omega/f_1)]}$$

$$I = \frac{\omega^3}{\frac{3k_2}{\omega} (\omega f_1^2 - f_1^3 \tan(\omega/f_1))}$$

W.R.T $k^2 = f_1/\omega \tan(\omega/f_1)$

$$I = \frac{\omega^3}{\frac{3\omega}{f_1 \tan(\omega/f_1)} [\omega f_1^2 - f_1^3 \tan(\omega/f_1)]}$$

$$I = \frac{\omega^2}{3 \left[\frac{\omega f_1^2}{f_1 \tan(\omega/f_1)} - \frac{f_1^3 \tan(\omega/f_1)}{f_1 \tan(\omega/f_1)} \right]}$$

$$I = \frac{\omega^2}{3 \left[\frac{\omega f_1}{\tan(\omega/f_1)} - f_1^2 \right]}$$

$$I = \frac{\omega^2}{3 \left[\frac{\omega f_1 - f_1^2 \tan(\omega/f_1)}{\tan(\omega/f_1)} \right]}$$

$$I = \frac{\omega^2 \tan(\omega/f_1)}{3 [\omega f_1 - f_1^2 \tan(\omega/f_1)]}$$

Divide by f_1^2

$$I = \frac{(\omega/f_1)^2 \tan^1(\omega/f_1)}{3 \left[(\omega/f_1) - \tan^1(\omega/f_1) \right]}$$

For commercial broadcasting, typical values are

$$f_1 = 2.1 \text{ kHz} \quad , \quad \omega = 15 \text{ kHz}$$

$$I = \frac{\left(\frac{15}{2.1}\right)^2 \tan^1\left(\frac{15}{2.1}\right)}{3 \left[\left(\frac{15}{2.1}\right) - \tan^1\left(\frac{15}{2.1}\right) \right]}$$

$$I = 4.2633$$

$$\text{In decibel} \Rightarrow I_{dB} = 10 \log_{10} 4.2633$$

$$I_{dB} = 6.3 \text{ dB}$$

thus by using the simple pre-emphasis and de-emphasis, a significant improvement in the noise performance of the receiver is obtained.